

Section 6.5

Logistic Growth

Partial Fraction Decomposition with Distinct Linear Denominators

If $f(x) = \frac{P(x)}{Q(x)}$, where P and Q are polynomials

with degree of P less than the degree of Q, and if Q(x) can be written as a product of distinct linear factors, then f(x) can be written as a sum of rational functions with distinct linear denominators.

Write the function $f(x) = \frac{x - 13}{2x^2 - 7x + 3}$ as a sum of rational functions with linear denominators

$$f(x) = \frac{A}{(2x-1)} + \frac{B}{(x-3)}$$
$$\frac{x-13}{2x^2-7x+3} = \frac{A(x-3) + B(2x-1)}{(2x-1)(x-3)}$$

$$\int \frac{x^2 - 9}{x} dx =$$

$$= \int \frac{x^2}{x} dx - \int \frac{9}{x} dx$$

$$= \int x dx - 9 \int \frac{1}{x} dx$$

$$= \frac{x^2}{2} - 9 \ln|x| + C$$

$$\int \frac{x}{x^2 - 9} dx = \frac{1}{2} \int \frac{1}{u} du$$

$$u = x^2 - 9 \quad = \frac{1}{2} \ln |x^2 - 9| + C$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$\frac{du}{2} = x dx$$

$$\int \frac{x+1}{x^2-9} dx =$$

$$(x+3)(x-3)$$

$$x+1 = A(x+3) + B(x-3)$$

$$\underline{\text{Let } x = -3}$$

$$-3+1 = 0 + B(-6)$$

$$-2 = -6B$$

$$\frac{1}{3} = B$$

$$\underline{\text{Let } x = 3}$$

$$3+1 = A(6) + 0$$

$$4 = 6A$$

$$\frac{2}{3} = A$$

$$\int \frac{2x - 7}{x^2 + 5x + 6} dx = \int \left(\frac{A}{x+3} + \frac{B}{x+2} \right) dx$$

(x+3)(x+2)

$$2x - 7 = A(x+2) + B(x+3)$$

$$\underline{\text{let } x = -2}$$

$$-4 - 7 = 0 + B$$

$$-11 = B$$

$$\underline{\text{let } x = -3}$$

$$-6 - 7 = A(-1) + 0$$

$$-13 = -A$$

$$13 = A$$

Find

$$\int \frac{3x^4 + 1}{x^2 - 1} dx =$$

$$\begin{array}{r} 3x^2 + 3 + \frac{4}{x^2 - 1} \\ x^2 + 0x - 1 \overline{) 3x^4 + 0x^3 + 0x^2 + 0x + 1} \\ \underline{- 3x^4 + 0x^3 + 3x^2} \\ 3x^2 + 0x + 1 \\ \underline{- 3x^2 + 0x + 3} \\ 4 \end{array}$$

Find the general solution to

$$\frac{dy}{dx} = \frac{6x^2 - 8x - 4}{(x^2 - 4)(x - 1)}$$

$$6x^2 - 8x - 4 = A(x-2)(x-1) + B(x+2)(x-1) + C(x+2)(x-2)$$

$$\underline{\text{Let } x=2}$$

$$6(4) - 16 - 4 =$$

$$4 = 4B$$

$$1 = B$$

$$\underline{\text{Let } x=1}$$

$$6 - 8 - 4 = C(3)(-1)$$

$$-6 = -3C$$

$$2 = C$$

$$\underline{\text{Let } x=-2}$$

$$6(4) + 16 - 4 = A(-4)(-3)$$

$$36 = 12A$$

$$A = 3$$

